Chapter 7
Linear Momentum & Center of Mass

Definition: momentum = mass X velocity

$$\boxed{\vec{p} = m\vec{v}} \quad \textit{where } \vec{p} = momentum$$

When we take the timed rate of change of $\vec{p}$

$$\frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$$

By Newton's Second Law $\sum\vec{F} = \vec{F}_{NET} = m\vec{a}$

Therefore $\boxed{\sum\vec{F} = \frac{\Delta\vec{p}}{\Delta t}}$ ← HOW NEWTON'S LAW WAS ORGINALLY WRITTEN

The principle of Conservation of Momentum

- ***Conserved means doesn't change***

☆ ***A system cannot change its momentum by itself.***

Unless a system is acted on by a NET external force the initial momentum of a system must equal the final momentum of a system. ← SOUND FAMILIAR?

However, two or more systems may exchange momentum. We will study how these changes occur.

Example Problem:

Water leaves a hose at the rate of 1.5 kg/s with a velocity of 20 m/s, and is aimed at the side of the car, which brings the water to rest. (We can assume no splash back occurs) What is the force exerted by the water on the car?

$$\Sigma F = \frac{\Delta p}{\Delta t} = \frac{m \Delta v}{\Delta t}$$

$$\Delta v = 20\,m/s - 0\,m/s$$

$$\frac{m}{\Delta t} = 1.5\,kg/s$$

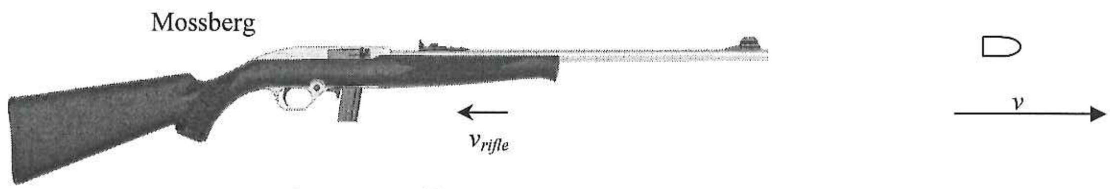
so $\Sigma F = (1.5\,kg/s)(20\,m/s)$

$$\therefore \boxed{\Sigma F = 30\,kg \cdot m/s}$$

Example Problem:

Conservation of Momentum – Collide & Stick together

A bullet whose mass, *m*, is 50.0g is fired horizontally with a speed, *v*, of 1,100 m/s by a rifle whose mass, M = 6.0 kg that is initially at rest. What is the speed of the rifle when it recoils?



$$P_i = P_f$$

$$m v_r + m v_B = m v'_R + m v'_B$$

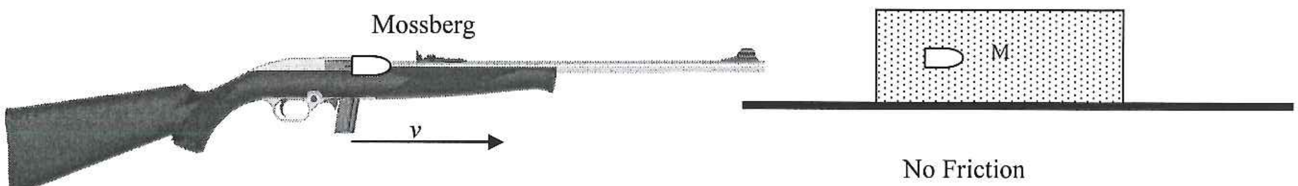
$$0 + 0 = (6\,kg)(v_R) + (.05\,kg)(1{,}100\,m/s)$$

$$\therefore \boxed{V_R = -9.1\overline{66}\,m/s}$$

Example Problem:

Conservation of Momentum – Collide & Stick together

A bullet whose mass, *m*, is 50.0g is fired horizontally with a speed, *v*, of 1,100 m/s into a large wooden block of mass, M = 6.0 kg that is initially at rest on a horizontal table. If the block is free to slide without friction across the table, what speed will it acquire after it has absorbed a bullet?



$$(mV)_B + (mV)_{BLOCK} = (mV)_{B+BLOCK}$$

$$.05\,kg(1100\,m/s) + 0 = (.05 + 6.0)\,V'_{B+BLOCK}$$

$$\boxed{V_{B+BLOCK} = 9.09\,m/s}$$

Example Problem:

Conservation of momentum – Collide & Stick together

A 10,000 kg railroad car traveling at a speed of 24.0 m/s strikes an identical railroad car that is at rest. If the cars lock together as a result of the collision, what is their common speed afterward?

$m_1 = 10{,}000\,kg$

$v_1 = 24\,m/s$

$m_2 = 10{,}000\,kg$

$v_2 = 0\,m/s$

BEFORE AFTER

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

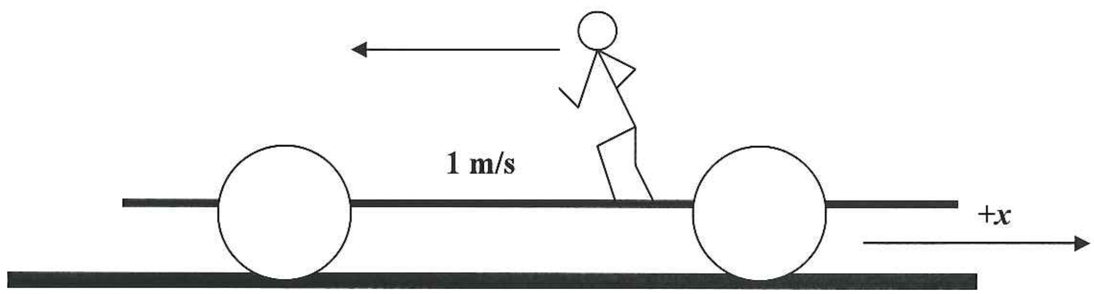
$$10{,}000(24) + 0 = 20{,}000 v'$$

$$\boxed{v' = 12\,m/s}$$

Example Problem:

Conservation of momentum – Internal Motion Problem

A man with a mass of 70 kg is standing on the front end of a flat railroad car, which has a mass of 1,000 kg and a length of 10 m. The railroad car is initially at rest relative to the track. The man then walks from one end of the car t to the other at a speed of 1.0 m/s relative to the track. Assume there is no friction in the wheels of the railroad car. (a) What happens to the cart while the man is walking? (b) How long does it take him to reach the other end of the car? (c) What happens when the man stops at the rear of the car?



a. THE CART WILL "RECOIL" AND MOVE THE OPPOSITE WAY THE MAN IS WALKING.

b. BEFORE AFTER

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$$0 + 0 = 70kg(-1\,m/s) + (1000kg)\,V_2'$$

$$V_2 = +.07\,m/s$$

SO $V_{\text{MAN WRT CART}} = -1\,m/s + .07\,m/s = -.93\,m/s$

$$V = \frac{\Delta x}{t} \rightarrow -.93\,m/s = \frac{-10m}{t} \rightarrow \therefore \boxed{t = 10.75\,sec}$$

c. THE CART ALSO STOPS.

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$$70(-1) + 1000(.07) = 0 + 1000(v_2') \rightarrow \boxed{V_2' = 0\,m/s}$$

1. (II) A child in a boat throws a 5.40-kg package out horizontally with a speed of 10.0m/s, (Fig. 7-28). Calculate the velocity of the boat immediately after, assuming it was initially at rest. The mass of the child is 26.0 kg and that of the boat is 55.0 kg.

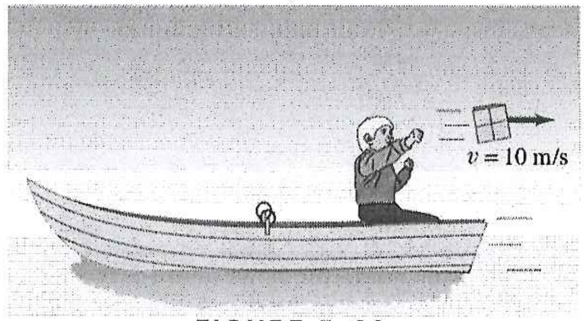


FIGURE 7-28

$+\hat{x}$ →

BEFORE AFTER

$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

$0 + 0 = 5.4(10) + 81(v_2')$

$54 = -81 v_2'$

$\therefore \boxed{v_2' = -.\overline{66}\ m/s}$

2. (II) Calculate the force exerted on a rocket, given that the propelling gases are expelled at a rate of 1300 kg/s with a speed of 40,000 m/s (at the moment of takeoff).

$\Sigma F = \frac{m \Delta v}{\Delta t} \rightarrow \Sigma F = (1300\ kg/s)(40,000\ m/s)$

$\frac{m}{t} = 1300\ kg/s$

$\boxed{\Sigma F = 5.2 \times 10^7\ N}$

$\Delta V = 40,000\ m/s - 0\ m/s$

3. (II) A 12,500-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0 m/s. A 5750-kg additional load is dropped into the car. What then will be the car's speed?

$$m_1V_1 + m_2V_2 = m_{1+2}V'_{1+2}$$

$$12500(18) + 0 = (12{,}500 + 5750)\,v'_{1+2}$$

$$225000 = 18250\,v'_{1+2}$$

$$\therefore \boxed{V' = 12.33 \, m/s}$$

4. (II) A gun is fired vertically into a 1.40-kg block of wood at rest directly above it. If the bullet has a mass of 21.0 g and a speed of 210 m/s, how high will the block rise into the air after the bullet becomes embedded in it?

$$m_1V_1 + m_2V_2 = m_{1+2}V'_{1+2}$$

$$.021(210) + 0 = (1.4 + .021)\,V'_{1+2}$$

$$V'_{1+2} = 3.103 \, m/s$$

$$\tfrac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$\tfrac{1}{2}(3.103)^2 = 9.8h$$

$V_i = 3.103 \, m/s$

$V_f = 0 \, m/s$

$g = -9.8 \, m/s$

$\Delta y = ?$

$$V_f^2 = V_i^2 + 2g\Delta y$$

$$0 = 3.103^2 + 2(-9.8)\,\Delta y$$

$$\boxed{\Delta y = .49m}$$

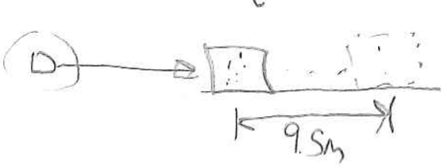
5. (II) A 15-g bullet strikes and becomes embedded in a 1.10-kg block of wood placed on a horizontal surface just in front of the gun. If the coefficient of kinetic friction between the block and the surface is 0.25, and the impact drives the block a distance of 9.5 m before it comes to rest, what is the muzzle speed of the bullet?

$$W = \Delta KE$$

$$-\mu mg\Delta x = \tfrac{1}{2}mV_f^2 - \tfrac{1}{2}mV_i^2$$

$$-.25(9.8)(9.5m) = -\tfrac{1}{2}V_i^2$$

$$V_i = 6.822 \, m/s$$



$$m_1V_1 + m_2V_2 = m_{1+2}V'_{1+2}$$

$$.015v_1 + 0 = 1.115(6.822)$$

$$.015v_1 = 7.6074$$

$$\therefore \boxed{V_1 = 507.16 \, m/s}$$

6. (II) An Atomic nucleus initially moving at 420 m/s emits an alpha particle in the direction of its velocity and the new nucleus slows to 350 m/s. If the alpha particle has a mass of 4.0 u and the original nucleus has a mass of 222 u, what speed does the alpha particle have when it is emitted?

BEFORE AFTER

$$m_1 v_1 = m_1' v_1' + m_2 + v_2$$

$$(222u)(420\,m/s) = (218u)(350\,m/s) + (4u)V_2$$

$$93240 = 76300 + 4V_2$$

$$\boxed{V_2 = 4235\,m/s}$$

7. (II) A 13-g bullet traveling 230 m/s penetrates a 2.0-kg block of wood and emerges going 170 m/s. If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$(.013)(230) + 2(0) = (.013)(170) + 2v_2'$$

$$2.99 = 2.21 + 2v_2'$$

$$\boxed{V_2 = .39\,m/s}$$

Definition of Impulse

RECALL: $\sum \vec{F} = \dfrac{\Delta \vec{p}}{\Delta t}$

Therefore: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \sum \vec{F}_{NET} \Delta t$

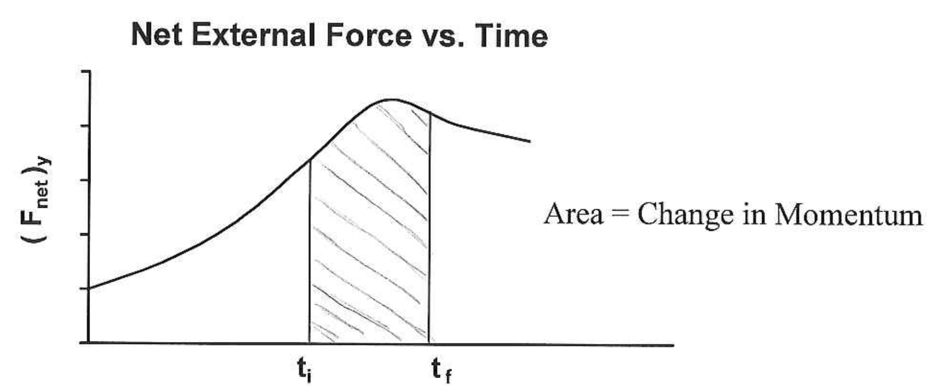
The impulse of the **external forces** acting on a system is equal to the change in the system's momentum

$\vec{J}$ = *IMPULSE*

$$\boxed{\vec{J} \equiv (\sum \vec{F}_{NET})\Delta t = \vec{p}_f - \vec{p}_i = \Delta\vec{p}}$$

$$\boxed{\vec{J} \equiv \int_{t_i}^{t_f} F(t)dt = \Delta\vec{p}}$$

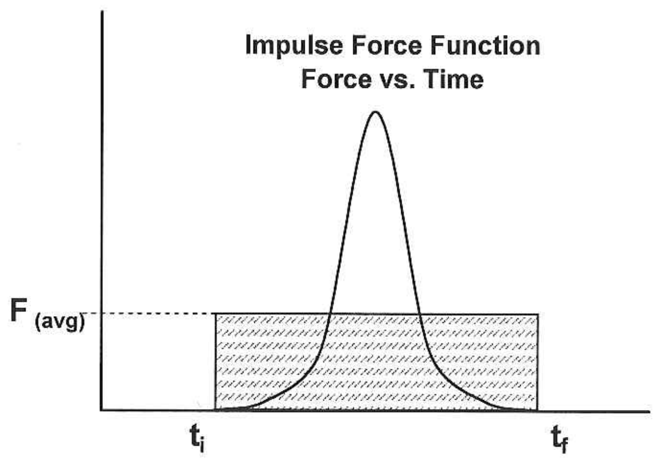
Area bounded by F(t) function and x-axis



IMPULSE AND COLLISIONS

Collision – When two or more objects come close together or hit and exert forces on each other for a short time

Impulse Forces – Forces that are exerted for a short time interval



The average Force over the interval Δt is:

$$\vec{F}_{avg} = \frac{1}{\Delta t}\left[\sum F_{net}\Delta t\right] = \frac{\vec{J}}{\Delta t}$$

F_{avg} is the constant force which gives the same impulse, $\vec{J}$, in the time interval Δt.

Example Problem: Impulse

A pitched 140 g baseball, in horizontal flight with a speed v_i of 39 m/s, is struck by a batter. After leaving the bat, the ball travels in the opposite direction with a speed v_f, also 39 m/s.

a. What impulse, $\vec{J}$, acts on the ball while it is in contact with the bat?

b. The impact time, Δt, for the baseball-bat collision is 1.2 ms, a typical value. What average force acts on the baseball?

$m = .14\,kg$

$V_i = 39\,m/s$

$V_f = -39\,m/s$

a. $\vec{J} = \Delta\vec{p} = m\Delta\vec{v}$

$J = (.14\,kg)(-39\,m/s - 39\,m/s)$

$\boxed{J = -10.92\,kg\,m/s}$

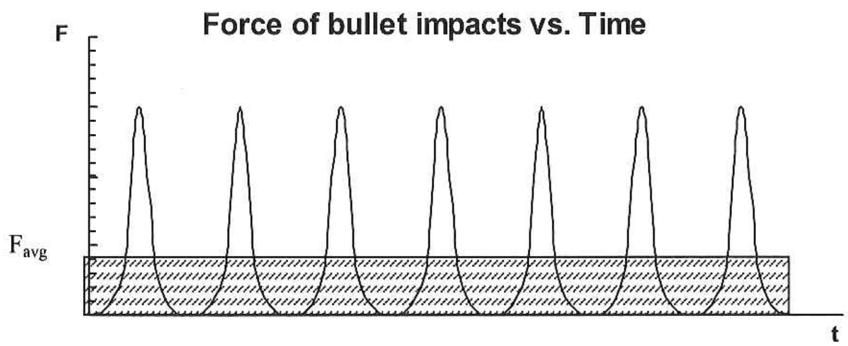
b. $J = F \cdot t$

$10.92\,kg\,m/s = F\,(.0012\,sec)$

$\boxed{F = 9100\,N}$

Example Problem: Series of collisions

A machine gun fires *R* bullets per second. Each bullet has mass, *m* and speed, *v*. The bullets strike a fixed target, where they stop. What is the average force exerted on the target over a time that is long compared with the time between bullets?
The graph shows the instantaneous force on the target.



$$F \cdot t = m\Delta V$$

$$\boxed{F_{Ave} = \frac{mv}{t}} \rightarrow \boxed{F_{Ave} = \frac{mv}{1/R}} = Rmv$$

8. (I) A tennis ball may leave the racket of a top player on the serve with a speed of 65.0 m/s. If the ball's mass is 0.0600 kg and it is in contact with the racket for 0.0300 s, what is the average force on the ball? Would this force be large enough to lift a 60-kg person?

$V = 65\,m/s$

$m = .06\,kg$

$t = .03\,s$

$F = ?$

$F \cdot t = m\Delta v$

$F = \frac{m\Delta v}{t}$

$F = \frac{.06(65\,m/s)}{.03}$

$\boxed{F = 130N}$

60 kg PERSON: $F_g = mg$

$F_g = 60(9.8)$

$F_g = 588N$

SO $\boxed{\text{No.}}$

9. (II) A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 5.0×10^{-3} s. Find (*a*) the impulse imparted to the golf ball and (*b*) the average force exerted on the ball by the golf club.

$m = .045\,kg$

$V = 45\,m/s$

$t = 5 \times 10^{-3}\,sec$

a. $J = F \cdot t = m\Delta v$

$J = .045\,kg(45\,m/s)$

$\boxed{J = 2.025\,kg \cdot m/s}$

b. $J = F \cdot t$

$2.025 = F(5 \times 10^{-3})$ $\boxed{F = 405N}$

$+\hat{x}$

10. (II) A tenn is ball of mass $m = 0.060$ kg and speed $v = 25$ m/s strikes a wall at a 45° angle and rebounds with the same speed at 45° (Fig 7-29). What is the impulse given to the wall?

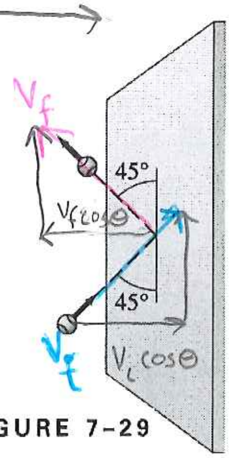


FIGURE 7–29

MOMENTUM AND IMPULSE ARE VECTORS!

$\vec{J} = \Delta\vec{p} = m\Delta\vec{V}$

ONLY THE HORIZONTAL COMPONENT HAS A CHANGE IN VELOCITY SO:

$\vec{J} = m\Delta V$

$\vec{J} = m(-V_f \cos\theta - V_i \cos\theta)$

$= .06\,kg(-25\cos 45 - 25\cos 45) \rightarrow \therefore \boxed{\vec{J} = -2.12\,kg\,m/s\,\hat{x}}$

11. (II) A 115 -kg fullback is running at 4.0 m/s to the east and is stopped in 0.75 s by a head-on tackle by a tackler running due west. Calculate (*a*) the original momentum of the fullback, (*b*) the impulse exerted on the fullback, (*c*) the impulse exerted on the tackler, and (*d*) the average force exerted on the tackler.

$m = 115\,kg$

$\Delta v = -4\,m/s$

$t = .75\,sec$

$+\hat{x} \longrightarrow$

a. $p = mv$

$= 115\,kg\,(+4\,m/s)$

$\boxed{\vec{p} = 460\,kg\,m/s\,\hat{x}}$

b. $J = \Delta p = 0\,kg\,m/s - 460\,kg\,m/s$

$\boxed{\vec{J} = -460\,kg\,m/s\,\hat{x}}$

c. $\boxed{460\,kg \cdot m/s\,\hat{x}}$ This is Newton's third law. The forces must be equal and opposite. The time is the same. So the impulse is equal and opposite.

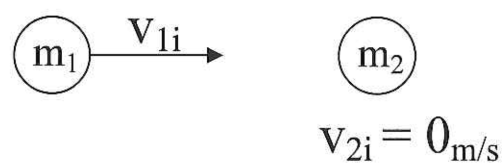
d. $\vec{J} = \vec{F} \cdot t$

$460\,kg \cdot m/s = F\,(.75)$

$\therefore \boxed{\vec{F} = 613.33\,N\,\hat{x}}$

ELASTIC COLLISIONS

Consider a 2 body system:

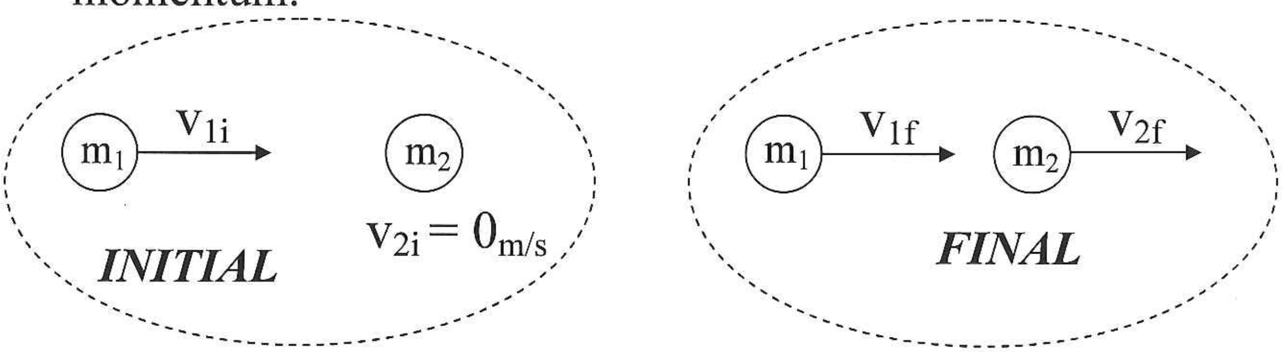


Assume <u>closed system</u> (no mass enters or leaves)
Assume <u>isolated system</u> (no external forces act on it)

The linear momentum, $\vec{p}$, is <u>always</u> conserved in a closed, isolated system, whether or not the collision is elastic, because the forces are all internal. (Internal forces cancel out because they act on the same object or system.)

If the kinetic energy of the system is also conserved then the collision is <u>elastic</u>.

Therefore, Elastic collisions conserve energy and momentum.



If elastic then:

$$\vec{p}_{i(system)} = \vec{p}_{f(system)} \qquad m_1\vec{v}_{1i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

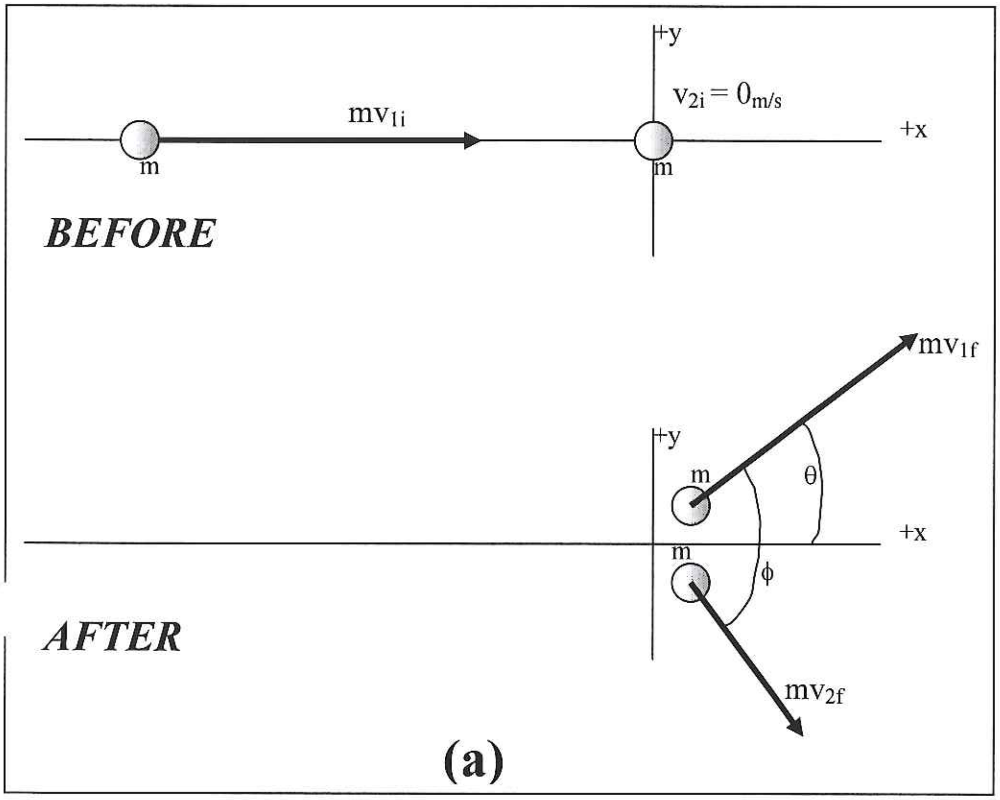
and

$$KE_i = KE_f \qquad \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

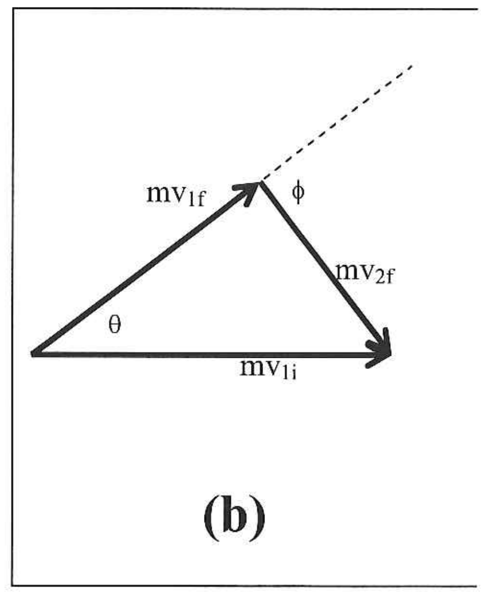
Example Problem:

Elastic Collisions – Equal mass, 90°, after collision problem

Two particles of equal masses have an elastic collision, the target particle being initially at rest. Show that (unless the collision is head-on) the two particles will always move off perpendicular to each other after the collision.



(a)



(b)

LOOK AT CONSERVATION OF ENERGY

$$\frac{1}{2}\cancel{m}v_{1i}^2 = \frac{1}{2}\cancel{m}v_{1f}^2 + \frac{1}{2}\cancel{m}v_{2f}^2$$

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

LOOK AT MOMENTUM
(NEED COMPONENTS SINCE VECTOR)

$$X: \cancel{m}v_{1ix} = \cancel{m}v_{1fx} + \cancel{m}v_{2fx}$$

$$Y: 0 = \cancel{m}v_{1fy} + \cancel{m}v_{2fy}$$

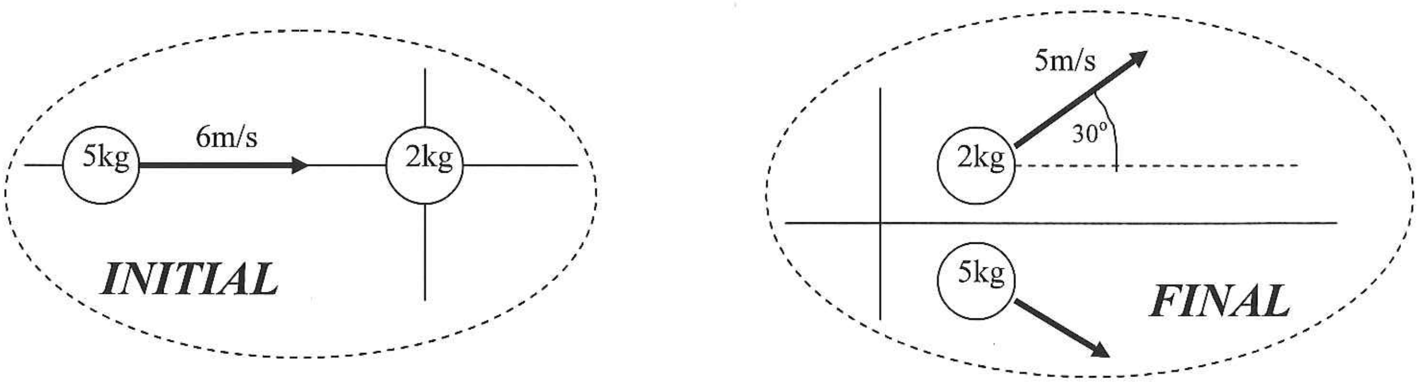
SO USING VECTOR ADDITION OF X momentum velocities AND KNOWING THAT THE Y COMPONENTS MUST EQUAL ZERO, OUR TRIANGLE LOOKS LIKE →

IN ORDER FOR THIS TO HOLD TRUE WITH $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$, THIS MUST BE A RIGHT TRIANGLE. (BECAUSE OF PYTHAGOREAN THEOREM)

Example Problem:
2-D Collision Problem – Elastic or Inelastic?

A 5 kg particle moves in the +x direction with a velocity of 6 m/s shown in the figure below. You are told that it makes an elastic collision with a 2 kg particle that is initially at rest. After the collision, the 2 kg particle moves with a speed of 5 m/s in the direction 30° above the x-axis, as shown in the figure below.

What are the x and y components of the 5 kg particle after the collision? Were you informed correctly? Is this an elastic collision?



CONSERVATION OF MOMENTUM:

$$X: \quad m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$5kg(6\,m/s) + 0 = 5kg(V_{1x}') + 2kg(5\,m/s)\cos 30$$

$$\boxed{\vec{V}_{1x}' = 4.268\,m/s\,\hat{x}}$$

$$Y: \quad 0 = m_1 V_1' + m_2 V_2'$$

$$0 = 5kg(V_{1y}') + 2kg(5\sin 30\,m/s)$$

$$\boxed{\vec{V}_{1y}' = -1\,m/s\,\hat{y}}$$

SO TOTAL $V' = \sqrt{4.268^2 + 1^2} \rightarrow V_1' = 4.38\,m/s$

IS ENERGY CONSERVED? $\frac{1}{2}m_1V_1^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2$

$$\frac{1}{2}(5)(6^2) = \frac{1}{2}(5)(4.38)^2 + \frac{1}{2}(2)(5^2)$$

$90 > 83$ SO $\boxed{\text{NOT ELASTIC}}$

Elastic Collisions: Target ball at rest

Will the incident ball rebound, stop or continue forward?

$$\vec{p}_{i(system)} = \vec{p}_{f(system)}$$

$$m_1 \vec{v}_{1i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$m_1 \left(\vec{v}_{1i} - \vec{v}_{1f} \right) = m_2 \vec{v}_{2f}$$

$$\frac{m_1^2}{m_2^2} \left(v_{1i} - v_{1f} \right)^2 = v_{2f}^2$$

$$KE_i = KE_f$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 \left(v_{1i}^2 - v_{1f}^2 \right) = m_2 v_{2f}^2$$

$$\frac{m_1}{m_2} \left(v_{1i} + v_{1f} \right)\left(v_{1i} - v_{1f} \right) = v_{2f}^2$$

$$\frac{m_1^{\cancel{2}}}{m_2^{\cancel{2}}} \left(v_{1i} - v_{1f} \right)^{\cancel{2}} = \cancel{\frac{m_1}{m_2}} \left(v_{1i} + v_{1f} \right) \cancel{\left(v_{1i} - v_{1f} \right)}$$

$$\frac{m_1}{m_2} \left(v_{1i} - v_{1f} \right) = v_{1i} + v_{1f}$$

$$\frac{m_1}{m_2} v_{1i} - v_{1i} = \frac{m_1}{m_2} v_{1f} + v_{1f}$$

$$v_{1i} \left(\frac{m_1}{m_2} - 1 \right) = v_{1f} \left(\frac{m_1}{m_2} + 1 \right)$$

IF $m_1 > m_2$, $\frac{m_1}{m_2} - 1$ IS POSITIVE, SO v_{1f} IS POSITIVE → CONTINUE FORWARD

IF $m_1 = m_2$, $\frac{m_1}{m_2} - 1$ IS ZERO, SO v_{1f} IS 0 → STOP

IF $m_2 > m_1$, $\frac{m_1}{m_2} - 1$ IS NEGATIVE, SO v_{1f} IS NEGATIVE → REBOUND

INELASTIC COLLISIONS

Inelastic Collision – a collision in which the kinetic energy of the system of colliding bodies is not conserved.

EXAMPLE: A ball being dropped to the ground and only rebounding to ½ its initial height is noticeably inelastic.

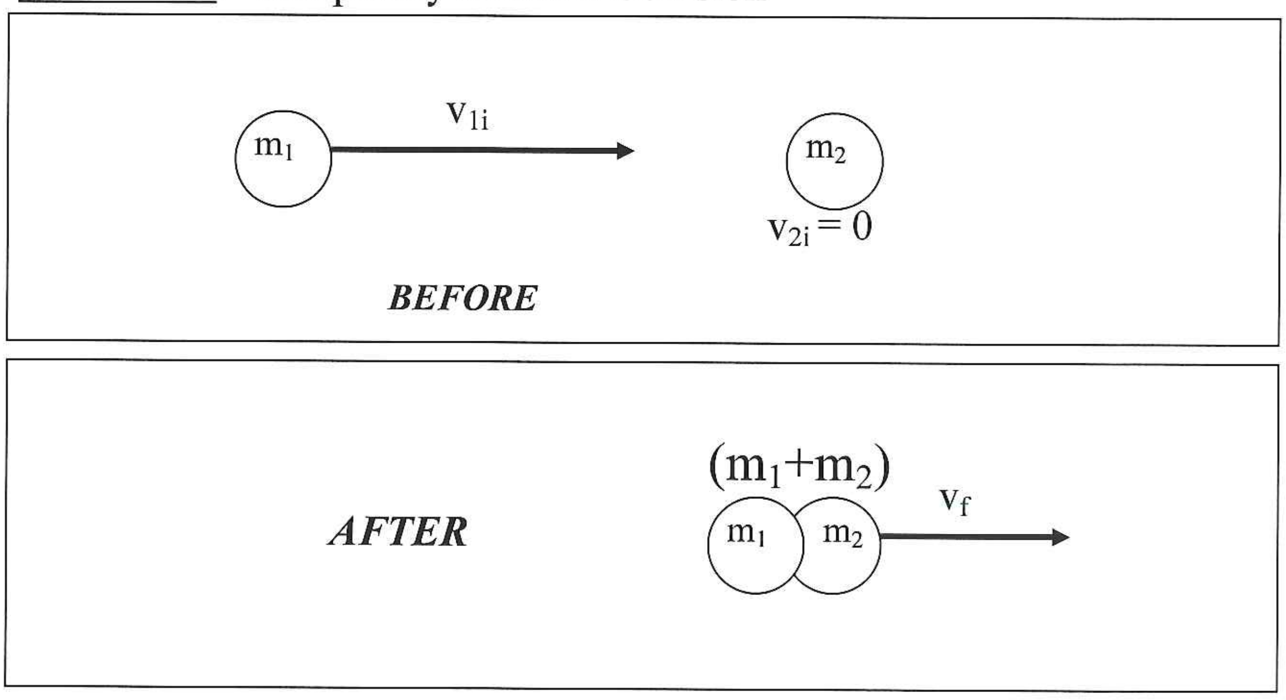
- The kinetic energy lost is transformed into some other form of energy, often thermal.

EXAMPLE: Putty dropped onto a floor does not rebound at all.

- Collisions where there is no rebound at all (the particles stick together) are called completely inelastic collisions.

NOTE: For completely inelastic collisions momentum is conserved, as long as the system is isolated and closed.

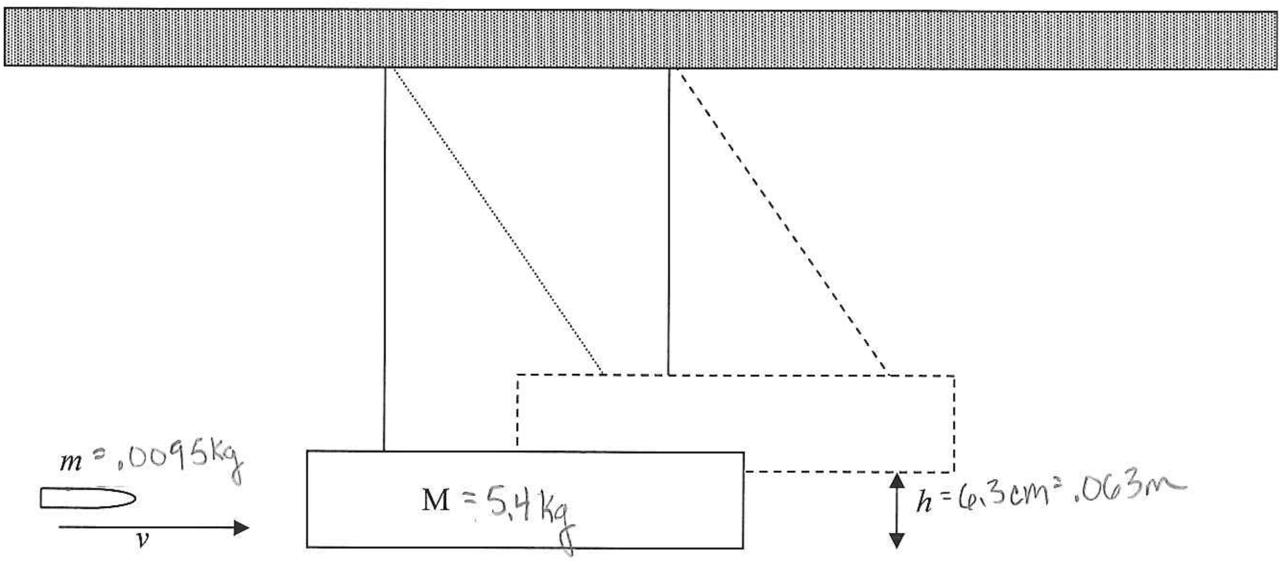
EXAMPLE: Completely Inelastic Collision



Example Problem:
Inelastic Collision Problem (Ballistic Pendulum)

A ballistic pendulum is a device that was used to measure the speed of bullets before electronic timing devices were developed. The device consists of a large block of wood of mass, M = 5.4 kg, hanging from two long cords. A bullet of mass, m = 9.5 g is fired into the block, coming quickly to rest. The *block + bullet* then swing upward, their center of mass rising a vertical distance, h = 6.3 cm before the pendulum comes momentarily to rest at the end of its arc.

a) What was the speed of the bullet just prior to the collision?
b) What is the initial kinetic energy of the bullet? How much of this energy remains as mechanical energy of the swinging pendulum?



a. FIRST FIND INITIAL SPEED OF BLOCK + BULLET: PE = KE

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$(9.8)(.063) = \frac{1}{2}v^2 \rightarrow v = 1.11 \, m/s$$

NOW USE Momentum: $m_B V_B + m_W V_W = (m_B + m_W)v'$

$$.0095\, V_B + \cancel{5.4(0)} = (.0095 + 5.4)(1.11\, m/s)$$

$$\boxed{V_B = 632.75\, m/s}$$

b. $KE = \frac{1}{2}(.0095)(632.75)^2 = 1901.77\,J$

$PE = (.0095 + 5.4)(9.8)(.063) = 3.34\,J$

SO 1898.4J LOST.

12. (II) A 0.45 0-kg ice puck, moving east with a speed of 3.00 m/s, has a head-on collision with a 0.900-kg puck initially at rest. Assuming a perfectly elastic collision, what will be the speed and direction of each object after the collision?

HEAD ON ASSUMES ONE DIMENSION

$m_1 = .45\,kg$

$V_{1i} = 3\,m/s$

$m_2 = .9\,kg$

$V_{2i} = 0\,m/s$

$m_1 V_{1i} + \cancel{m_2 V_{2i}}^{\,0} = m_1 V_{1f} + m_2 V_{2f}$

$.45(3) = .45 v_{1f} + .9 v_{2f} \rightarrow \dfrac{1.35 - .45 V_{1f}}{.9} = V_{2f}$

ELASTIC SO KE CONSERVED

$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_2 V_{2f}^2 + \frac{1}{2} m_1 V_{1f}^2$

$\frac{1}{2}(.45)(3)^2 = \frac{1}{2}(.9) V_{2f}^2 + \frac{1}{2}(.45) V_{1f}^2$

$2.025 = .45 V_{2f}^2 + .225 V_{1f}^2 \rightarrow V_{2f} = \sqrt{\dfrac{2.025 - .225 V_{1f}^2}{.45}}$

GRAPHING CALC $\rightarrow \boxed{V_{1f} = -1\,m/s}$

$\therefore V_{2f} = \dfrac{1.35 - .45(-1)}{.9} \rightarrow \boxed{V_{2f} = 2\,m/s}$

13. (II) A pair of bu mper cars in an amusement park ride collide elastically as one approaches the other directly from the rear (Fig. 7-31). One has a mass of 450 kg and the other 550 kg, owing to differences in passenger mass. If the lighter one approaches a 4.50 m/s and the other is moving at 3.70 ms/, calculate (*a*) their velocities after the collision, and (*b*) the change in momentum of each.

(a)

(b)

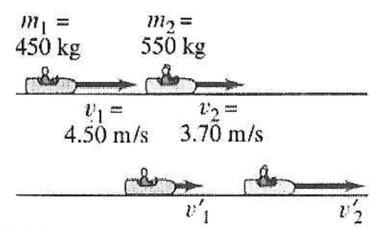


FIGURE 7–31
(a) before collision, (b) after collision.

$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

$450(4.5) + 550(3.7) = 450 v_1' + 550 v_2'$

$4060 = 450 v_1' + 550 v_2' \rightarrow v_2' = \dfrac{4060 - 450 v_1'}{550}$

KE conserved: $\frac{1}{2}(450)(4.5)^2 + \frac{1}{2}(550)(3.7)^2 = \frac{1}{2}(450) v_1'^2 + \frac{1}{2}(550) v_2'^2$

$8321 = 225 v_1'^2 + 275 v_2'^2$

$v_2' = \sqrt{\dfrac{8321 - 225 v_1'^2}{275}}$

Graphing calc $\rightarrow$

a. $\boxed{v_2' = 5.468\,m/s \quad v_1' = 2.338\,m/s}$

b. $\Delta\vec{p} = 450(2.338 - 4.5) = \boxed{-972.9\,kg\,m/s}$

$\Delta\vec{p} = 550(5.468 - 3.7) = \boxed{972.4\,kg\,m/s}$

$\vec{p}$ conserved in an closed, isolated system

14. (III) In a physics lab, a cube slides down a frictionless incline as shown in Fig. 7-32, and elastically strikes a cube at the bottom that is only one-half its mass. If the incline is 30 cm high and the table is 90 cm off the floor, where does each cube land?

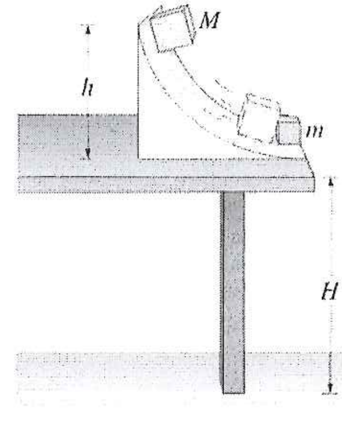


FIGURE 7-32

$M = 2m$ ① First find initial velocity before collision:

$h = .3m$ $\quad \cancel{M}gh = \frac{1}{2}\cancel{M}v^2$

$H = .9$ $\quad 9.8(.3) = \frac{1}{2}v^2 \rightarrow v = 2.4249\,m/s$

② now find final velocities (launch speed)

$\vec{p}: \cancel{M}v_1 = \cancel{M}v_1' + \frac{1}{2}\cancel{M}v_2'$ $\quad$ KE: $\cancel{\frac{1}{2}}\cancel{M}v_1^2 = \cancel{\frac{1}{2}}\cancel{M}v_1'^2 + \frac{1}{2}(\cancel{\frac{1}{2}}\cancel{M})v_2'^2$

$2.4249 = v_1' + \frac{1}{2}v_2'$ $\quad 2.4249^2 = v_1'^2 + \frac{1}{2}v_2'^2$

$v_1' = 2.4241 - .5v_2'$ $\quad v_1' = \sqrt{2.4249^2 - .5v_2'^2}$

calc graph

$v_1' = .8083\,m/s$

$v_2' = 3.2332$

③ now use kinematics.

time for both is same:

$\Delta y = \cancel{v_{iy}t}^{\,0} + \frac{1}{2}a_y t^2$

$-.9 = -4.9t^2 \rightarrow t = .42857\,sec$

$\Delta x = v_{ix}t$

$= .8083(.42859) = \boxed{.346m}$

$\Delta x = v_{ix}t$

$= 3.2332(.42857)$

$= \boxed{1.3857m}$

15. (II) An 18 -g rifle bullet traveling 230 m/s buries itself in a 3.6-kg pendulum hanging on a 2.8-m-long string, which makes the pendulum swing upward in an arc. Determine the horizontal component of the pendulum's displacement.

$m_B = .018kg$ $\quad m_B v_B = (m_B + m_P)V_P$

$V_B = 230\,m/s$ $\quad .018(230) = (.018 + 3.6)V_P$

$m_P = 3.6kg$ $\quad V_P = 1.144\,m/s$

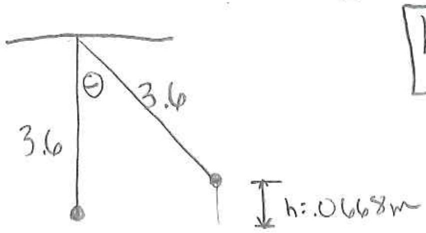
SO FIND HOW HIGH IT GOES:

KE = PE

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$\frac{1}{2}(1.144)^2 = 9.8h$

$\boxed{h = .0668m}$



16. (II) An explosio n breaks an object into two pieces, one of which has 1.5 times the mass of the other. If 7500 J were released in the explosion, how much kinetic energy did each piece acquire?

Conservation of $\vec{p}$: $0 = m_1 v_1 + m_2 v_2$

$0 = \cancel{m_1} v_1 + 1.5 \cancel{m_1} v_2 \rightarrow v_1 = -1.5 v_2$

$KE_1 = \frac{1}{2} m_1 v_1^2$

$KE_2 = \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2}(1.5 m_1)\left(-\frac{v_1}{1.5}\right)^2$

$= \frac{1}{2} m_1 v_1^2 \left(\frac{1}{1.5}\right)$

$= \frac{2}{3}\left[\frac{1}{2} m_1 v_1^2\right] = \frac{2}{3} KE_1$

Total $E = KE_1 + \frac{2}{3} KE_1 = \frac{5}{3} KE_1$

$7500 = \frac{5}{3} KE_1 \rightarrow \boxed{KE_1 = 4500 J}$ so $\boxed{KE_2 = 3000 J}$

17. (II) A 1.0 x 10 ^{3}-kg Toyota collides into the rear end of a 2.2 x 10^3-kg Cadillac stopped at a red light. The bumpers lock, the breaks are locked, and the two cars skid forward 2.8 m before stopping. The police officer, knowing that the coefficient of kinetic friction between tires and road is 0.40, calculates the speed of the Toyota at impact. What is that speed? HOW MUCH KE LOST?

$W_{fr} = \Delta KE$

$-\mu \cancel{m} g \Delta x = \cancel{\frac{1}{2} m v_f^2}^{0} - \frac{1}{2} \cancel{m} v_i^2$

$-.4(9.8)(2.8) = -\frac{1}{2} v_i^2$

$v_i^2 = 4.685 m/s$

$m_T v_T + \cancel{m_c v_c}^{0} = m_{T+C} v_{T+C}$

$(1.0 \times 10^3) v_T = (1 \times 10^3 + 2.2 \times 10^3)(4.685)$

$\boxed{v_T = 14.99 m/s}$

INITIAL KE: $\frac{1}{2} m_T v_T^2$

$\frac{1}{2}(1 \times 10^3)(14.99)^2$

$112,394.24 J$

FINAL KE: $\frac{1}{2}(m_T + m_c) v_f^2$

$\frac{1}{2}(1 \times 10^3 + 2.2 \times 10^3)(4.685)^2$

$35118.76 J$

$\therefore$ $\boxed{77,275.48 J \text{ LOST}}$